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Time : 55 minutes
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MATHEMATICS 218
QUIZ 2

NAME
ID#

Circle your section number :

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
8 M	11 M	2 M	12 M	1 M	4 M	9 F	11 M	11 F	2 T	3:30 T	5 T	1 M	3 M	4 M

PROBLEM GRADE

PART I

1 ------- / 18

2 ------- / 22

3 ------- / 10

4 ------- / 8

PART II

5	6	7	8	9	10
a	a	a	a	a	a
b	b	b	b	b	b
c	c	c	c	c	c
d	d	d	d	d	d
e	e	e	e	e	e

5-10 ------- / 24

PART III

11	12	13	14	15	16	17	18	19
T	T	T	T	T	T	T	T	T
F	F	F	F	F	F	F	F	F

11-19 ------- / 18

TOTAL ------- / 100.

PART I. Answer each of the following problems in the space provided for each problem (Problem 1 to Problem 4).

1. Let $A=\begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 4 & 2 & 5 \end{pmatrix}$

(a) Find a basis of the null space N(A).
(b) Find a basis of the column space Col(A).

[18 points]

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 4 & 2 & 5 \end{pmatrix} \xrightarrow{-2,\,-4} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$col(A) = span\left\{ \underbrace{\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}}_{\text{lin indep}} \right\}$$

$$\Rightarrow \left\{ \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} \right\} \text{ forms a basis for } col(A)$$

<u>for N(A)</u>
solve $Ax=0 \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$$\Rightarrow \begin{array}{l} x+z=0 \\ 2y+z=0 \end{array} \Rightarrow \begin{array}{l} x=-z=2y \\ z=-2y \end{array}$$

for $y=t$ $\quad x=2t$, $z=-2t$

$$N(A) = \left\{ \begin{pmatrix} 2t \\ t \\ -2t \end{pmatrix} / t \in \mathbb{R} \right\} = span\left\{ \underbrace{\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}}_{\text{lin indep}} \right\}$$

$$\Rightarrow \left\{ \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \right\} \text{ basis for } N(A)$$

2.

(a) Let **U** be the subset of $\mathbb{R}^3$ defined by:

$$U=\left\{\begin{pmatrix}x\\2x\\z\end{pmatrix}\in R^3 \;\middle|\; x,z\in R\right\}$$

Show that U is a <u>subspace</u> of $\mathbb{R}^3$ <u>and find a basis for</u> U

[10 points]

① $U\neq\emptyset$

since $\begin{pmatrix}0\\0\\0\end{pmatrix}\in U$

② if $\begin{pmatrix}x\\2x\\z\end{pmatrix}, \begin{pmatrix}m\\2m\\n\end{pmatrix}\in U$ then $\begin{pmatrix}x\\2x\\z\end{pmatrix}+\begin{pmatrix}m\\2m\\n\end{pmatrix}=\begin{pmatrix}x+m\\2(x+m)\\z+n\end{pmatrix}\in U$

③ if $k\in\mathbb{R}$, $\begin{pmatrix}x\\2x\\z\end{pmatrix}\in U$

then $k\begin{pmatrix}x\\2x\\z\end{pmatrix}=\begin{pmatrix}kx\\2(kx)\\z\end{pmatrix}\in U$

<u>Basis for U</u>?

let $u\in U\Rightarrow u=\begin{pmatrix}x\\2x\\z\end{pmatrix}=\begin{pmatrix}x\\2x\\0\end{pmatrix}+\begin{pmatrix}0\\0\\z\end{pmatrix}$

$=x\begin{pmatrix}1\\2\\0\end{pmatrix}+z\begin{pmatrix}0\\0\\1\end{pmatrix}$

$\Rightarrow U=\text{span}\left\{\underbrace{\begin{pmatrix}1\\2\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}}_{\text{lin indep}}\right\}$

$\Rightarrow\left\{\begin{pmatrix}1\\2\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$ form a basis for U.

2(b) Let **W** be the subspace of R^3 given by $\mathbf{W}=\left\{\begin{pmatrix} s \\ t \\ t \end{pmatrix} \in R^3 \;\middle|\; s.t \in R\right\}$. Find a basis for

U∩W.

[7 points]

let $u \in U\cap W \Rightarrow u = \begin{pmatrix} a \\ b \\ c \end{pmatrix} / u \in U \;\&\; u \in W$

$\Rightarrow b = 2a \quad c = b$

$\Rightarrow c = b = 2a$

$\Rightarrow u = \begin{pmatrix} a \\ 2a \\ 2a \end{pmatrix} \Rightarrow U\cap W = \text{span}\left\{\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}\right\}$

lin indep

$\Rightarrow \left\{\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}\right\}$ basis for $U\cap W$

2(c) Let **V**=Span $\left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}\right\}$. Find a basis for **U∩V.**

[5 points]

$V = \text{span}\left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}\right\} = \text{span}\left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right\}$

$U = \text{span}\left\{\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right\}$

Since $\left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right\}$ lin indep $\Rightarrow U\cap V = \{0_{\mathbb{R}^3}\}$

$\Rightarrow S = \phi$ basis

or

let $u = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \in U\cap V$

$\Rightarrow u \in U \;\&\; u \in V$

$\Rightarrow u = \begin{pmatrix} a \\ b \\ c \end{pmatrix} / b = 2a \;\&\; a = b = c$

$a = b \Rightarrow a = 2a$

$\Rightarrow \boxed{a = 0}$

$\& \boxed{c = 0}$

$\Rightarrow U\cap V = \left\{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\right\} = \{0_{\mathbb{R}^3}\}$

$\Rightarrow S = \phi$ basis for $U\cap V$

3. Show that if $\{ \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3 \}$ is a basis for a vector space V, then every vector $\mathbf{v}$ in V is uniquely written as a linear combination $\mathbf{v} = c_1\mathbf{u}_1 + c_2\mathbf{u}_2 + c_3\mathbf{u}_3$, c_1, c_2, c_3 in $\mathbf{R}$.

[10 points]

suppose $v = c_1u_1 + c_2u_2 + c_3u_3$
$\& \; v = k_1u_1 + k_2u_2 + k_3u_3$

$\Rightarrow c_1u_1 + c_2u_2 + c_3u_3 = k_1u_1 + k_2u_2 + k_3u_3$

$\Rightarrow (c_1 - k_1)u_1 + (c_2 - k_2)u_2 + (c_3 - k_3)u_3 = 0$

$\{u_1, u_2, u_3\}$ basis $\Rightarrow \{u_1, u_2, u_3\}$ indep

$\Rightarrow \begin{cases} c_1 - k_1 = 0 \\ c_2 - k_2 = 0 \\ c_3 - k_3 = 0 \end{cases} \Rightarrow \begin{matrix} c_1 = k_1 \\ c_2 = k_2 \\ c_3 = k_3. \end{matrix}$

$\Rightarrow$ v has a unique representatn wr to the vectors u_1, u_2, u_3

4. Let $\mathbf{V}$ be a vector space with dim $\mathbf{V}$=3. Let $\mathbf{u}_1$ be a nonzero vector in V. Let $\{\mathbf{u}_1, \mathbf{w}_1, \mathbf{w}_2\}$ be a basis of $\mathbf{V}$. Let U=Span $\{\mathbf{u}_1\}$, and W=Span $\{\mathbf{w}_1, \mathbf{w}_2\}$. Show that $U \cap W = \{\mathbf{0}\}$.

[8 points]

let $v \in U \cap W$

$\Rightarrow v \in U \; \& \; v \in W$

$\Rightarrow v = k_1u_1 \; \& \; v = c_1w_1 + c_2w_2$

$\Rightarrow k_1u_1 = c_1w_1 + c_2w_2$

$\Rightarrow k_1u_1 - c_1w_1 - c_2w_2 = 0$

$\{u_1, w_1, w_2\}$ basis for $V \Rightarrow \{u_1, w_1, w_2\}$ indep

$\Rightarrow k_1 = 0 \; c_1 = 0 \; c_2 = 0$ is the only sol

$\Rightarrow v = 0u_1 = 0$

$\Rightarrow U \cap W = \{0\}$

PART II. Circle the correct answer for each of the following problems (Problem 5 to Problem 10) IN THE TABLE OF THE FRONT PAGE. [4 points for each correct answer]. NO penalty in this Part

5. Let A be a 4×4 matrix such that dim Col(A)=3, then

a. AX=**0** has a unique solution X=**0**.
b. A is invertible.
c. $|A| = 0$
d. All columns of A are linearly independent
e. none of the above.

[4 points]

6. Let S be the space of all symmetric 3×3 matrices A= (a_{ij}) such that $a_{12} = 0$. Then dim S is equal to:

(a) 9
(b) 3
(c) 6
(d) 5
(e) None of the above

[4 points]

7. The subspace of R^3 spanned by S={ $\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}2\\4\\6\end{pmatrix}, \begin{pmatrix}3\\6\\9\end{pmatrix}$ } has a dimension equal to

(a) 1
(b) 2
(c) 4
(d) 3
(e) none of the above.

[4 points]

8. Let V=M_{2x2} . Which one of the following statements is <u>False</u>:
   (a) Any set of 5 matrices in V is linearly dependent
   (b) Any linearly independent set of 4 matrices in V forms a basis for V
   (c) Any set of 4 matrices in V that spans V forms a basis for V
   (d) Any set of 3 matrices in V can be enlarged to a basis for V
   (e) dim V=4

[4 points]

9. Let V={ $p(x)\in P_4 : p(0)=0$ *and* $p''(x)=0\}$. *Then* dim V=

(a) 1
(b) 2
(c) 3
(d) 4
(e) none of the above.

[4 points]

10. Let T : **R** → **R** be a linear transformation such that $T(\sqrt{2})=8$. Then $T(5)=$

(a) $20\sqrt{2})$
(b) $8\sqrt{2})$
(c) $40\sqrt{2})$
(d) 5/8
(e) none of the above.

[4 points]

PART III Answer **TRUE** or **FALSE** only <u>IN THE TABLE IN THE FRONT PAGE</u>
(2 points for each correct answer, and -1 point <u>penalty</u> for each wrong answer)

11. Let V and W be the subspaces of $\mathbf{R}^3$ defined by:

$$V=\left\{\begin{pmatrix} a \\ 0 \\ a \end{pmatrix} \in R^3 : a \in R\right\},\quad W=\left\{\begin{pmatrix} c \\ b \\ c \end{pmatrix} \in R^3 \middle|\ c.b \in R\right\}$$

Then dim(V+W)=3.

12. If A is a 3×3 matrix such that $A^2=I$, then the Col (A) $=R^3$.

13. Let **U**, **V**, and **W** be subspaces of R^3 such that **U**+**V**=**U**+**W**. Then **V**=**W**

14. Let **U**, **W** be subspaces of $\mathbf{R}^3$ such that dim(**U**∩**W**)=dim(**W**), then **U**∩**W**=**W**.

15. If T:V→W is a linear map, and if $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ are linearly dependent vectors in V, then T($\mathbf{u}_1$), T($\mathbf{u}_2$), T($\mathbf{u}_3$) are linearly dependent in W.

16. Let T: $\mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation. If S is a subspace of $\mathbb{R}^3$ then $\{x \in \mathbb{R}^2 : T(x) \in S\}$ is a subspace of $\mathbb{R}^2$.

17. Let A be a fixed nxn matrix. The set S=$\{x \in \mathbb{R}^n : Ax = 2x\}$ is a subspace of $\mathbb{R}^n$.

18. Let W=$\left\{\begin{pmatrix} a & 0 \\ b & a+b \end{pmatrix} \in M_{2\times 2} \middle|\ a, b \in R\right\}$, then W is a subspace of $M_{2\times 2}$.

19. $\mathbb{R}^2$ has only one subspace of dimension 1.

[18 points]